

## Practice Problems on Limits and Continuity

1 A tank contains 10 liters of pure water. Salt water containing 20 grams of salt per liter is pumped into the tank at 2 liters per minute.

1. Express the salt concentration  $C(t)$  after  $t$  minutes (in g/L).
2. What is the long-term concentration of salt, i.e.,  $\lim_{t \rightarrow \infty} C(t)$ ?

2 Find the values of  $a$  and  $b$  that make  $f(x)$  continuous for all real  $x$ .

$$f(x) = \begin{cases} be^x + a + 1, & x \leq 0 \\ ax^2 + b(x + 3), & 0 < x \leq 1 \\ a \cos(\pi x) + 7bx, & x > 1 \end{cases}$$

3 Sketch the graph of a function  $f$  with the following properties:

- $\lim_{x \rightarrow 1} f(x) = 2$ , but  $f(1) = 1$
- $\lim_{x \rightarrow 3} f(x) = +\infty$
- $\lim_{x \rightarrow 2^+} f(x) = -1$ ,  $\lim_{x \rightarrow 2^-} f(x) = 3$
- $\lim_{x \rightarrow +\infty} f(x) = -2$
- $\lim_{x \rightarrow -\infty} f(x) = -\infty$

4 Show that the equation  $\sqrt{x-5} = \frac{1}{x+3}$  has at least one real solution.

5 Consider the rational function

$$f(x) = \frac{x^5 - x^4 - 2x^3}{x^4 - 3x^3 - x^2 + 3x}$$

- For what values of  $a$  does  $f$  have a removable discontinuity at  $a$ ? What is  $\lim_{x \rightarrow a} f(x)$  at those  $a$ ?
- For what values of  $a$  does  $f$  have an infinite discontinuity at  $a$ ?
- What is  $\lim_{x \rightarrow +\infty} f(x)$ ?

(Hint: Factor the numerator and the denominator.)

6 Find the value of  $a$  such that

$$\lim_{x \rightarrow -1} \frac{2x^2 - ax - 14}{x^2 - 2x - 3}$$

exists. What is the value of the limit?